Application Note

PN9000 Automated Phase Noise Measurement System Application Note #1

1 INTRODUCTION

Today, noise measurements have become an essential factor in the characterization of signals. This is easy to understand when one realizes that it is phase noise that:

- limits the operating range of radar
- degrades the quality of television pictures
- limits the precision of satellite positioning
- spoils the quality of data transmission

The primary characteristic of noise is its randomness, and this is due to the physical mechanisms which generate it.

Three leading types of noise are to be found in all electronic systems:

- Thermal Noise: random motion of the carriers in a conductor.
- Shot Noise: random flow of the carriers through a potential barrier.
- Flicker Noise: its origin is not well known. It seems to come from the macroscopic defects of the materials.

To characterize these noise sources, one must refer to the Theory of Random Processes. The mathematical tools will be simplified by making three assumptions according to the statistical process, i.e. on whether it is:

- Stationary (zero mean value)
- Ergodic (its statistical mean values are the same as its time mean values)
- Gaussian (its amplitude has a Gaussian distribution)

NOTE: Other random disturbances, such as random jumps - which do not correspond to the above assumptions, can add themselves to the above type of noise and alter noise measurement results.

1.1 TIME DOMAIN

The study of noise in relation to time gives a "random function". Noise can disturb any physical parameter - this function can therefore apply to voltage (Volts), current (Amps), phase (radians), frequency (Hertz), time (seconds), etc.

This function *x(t)* can be characterized by its distribution *p(x),* where each x value represents the probability that *x(t)=x*.

It is only possible to describe this time function statistically. According to the previous assumptions, *p(x)* is a Gaussian function (see Figure 1)

$$p(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{-x^2/2\sigma^2}$$

where σ is the standard deviation of *p(x)*.

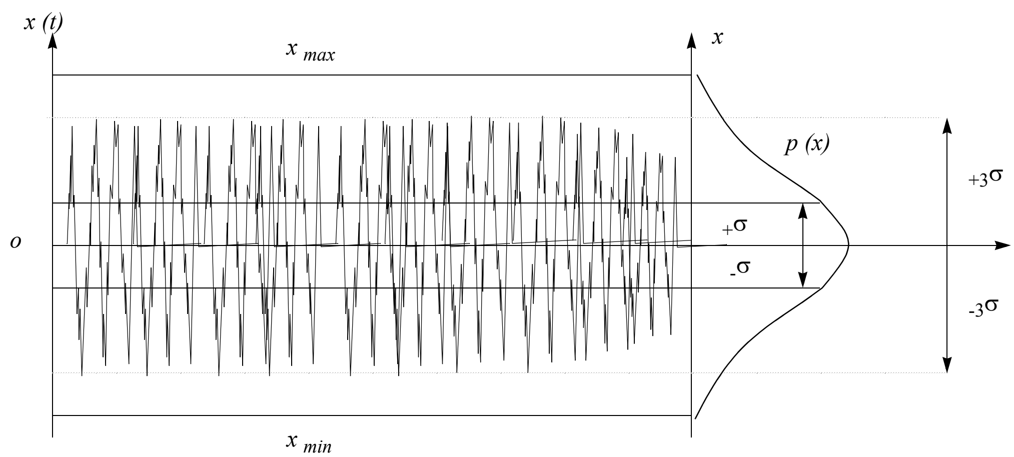

Figure 1

- The Gaussian function $p(x)$ can be completely characterized by one parameter σ.
- It can be calculated by means of a statistical average or a time average

rms value of $x(t) = \sqrt{\overline{x^2(t)}}$, assuming that the noise is ergodic $\sigma = \sqrt{\overline{x2(t)}}$

- p(x) is symmetrical $|x_{max}| = |x_{min}|$
- 99.7% of the function x(t) is located in the interval ± 3 σ. The value ± 3 σ can represent the peak-to-peak value of the noise. There is no absolute definition of peak-to-peak noise: ± 4 σ = 99.98% of *x(t)*.

- This statistical-time representation is useful to calculate bit-error rate. The error function *erf (x),* which is obtained in bit-error calculations, comes from the integration of *p(x)*. For some point x_o, the probability that $x(t) > x_o$ is

$$\int_{x_o}^{\infty} p(x)dx = \frac{1}{2}\left[1 - \int_{-x_o}^{x_o} p(x)dx\right] = \frac{1}{2}\left[1 - erf\left(\frac{x_o}{\sigma}\right)\right]$$

1.2 FREQUENCY DOMAIN

The basic parameter is the distribution of the noise power as a function of frequency. $S_x(f)$ is called the spectral density of phase fluctuations, frequency fluctuations, amplitude fluctuations, etc.

When one looks into the disturbances that can occur on a signal like phase noise, frequency noise, or amplitude noise, $S_x(f)$ is a low frequency, continuous spectrum (no discontinuity according to the frequency), defined for the positive frequencies (single-sideband spectrum). See Figure 2 below.

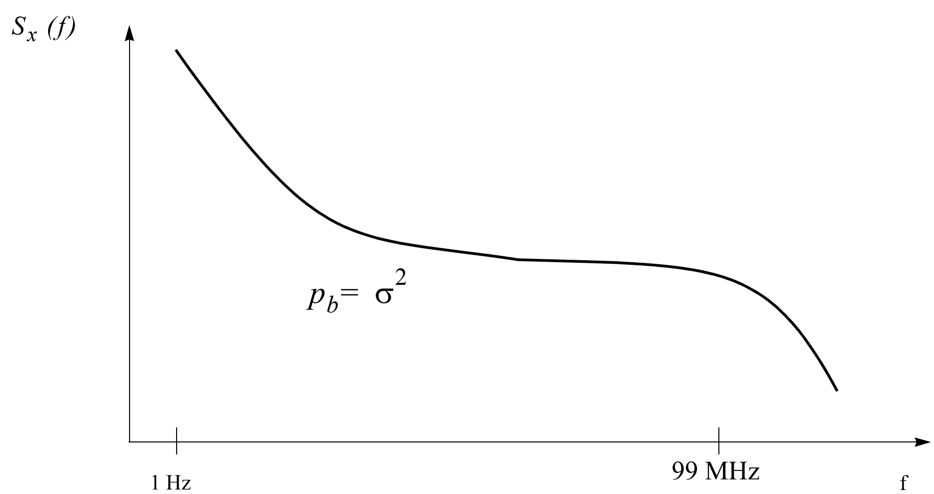

Figure 2

Units:

Phase Noise	x = radian	$S_x(f)$ = (radian)^2/Hz
Frequency Noise	x = Hertz	$S_x(f)$ = (Hertz)^2/Hz
Amplitude Noise	x = Volt	$S_x(f)$ = (Volt)^2/Hz

1.3 RELATIONSHIP BETWEEN THE TIME AND FREQUENCY DOMAINS

Noise power is the main common parameter of the two domains:

Time Domain (Figure 1)

x(t) is defined by its rms value $\sqrt{\overline{x^2(t)}}$

other notations $\sqrt{\langle x^2 \rangle}$, x_{rms}

p(x) is defined by σ (standard deviation)

Frequency domain (Figure 2)

$S_x(f)$ enables computation of total noise power $P_b = \int_0^\infty S_x(f)df$

The relationship between the two domains is $P_b = \overline{x^2(t)} = \sigma^2$

1.4 CHARACTERIZATION OF A SIGNAL

Noise, which has just been described, can affect a carrier signal (i.e. a signal which has a significantly higher level than the noise affecting it). There are four different ways of characterizing this signal:

SPECTRAL PURITY

The spectrum of an ideal sinusoidal signal $v(t)=A\sin[2\pi f_o t]$ corresponds to a DIRAC function at frequency f_o

$$S_v(f) = \frac{A^2}{2\sigma(f-f_o)} \text{ Watt/Hz}$$

The disturbances due to the frequency and amplitude fluctuations give rise to a spectral bandwidth and noise sidebands, Figure 3.

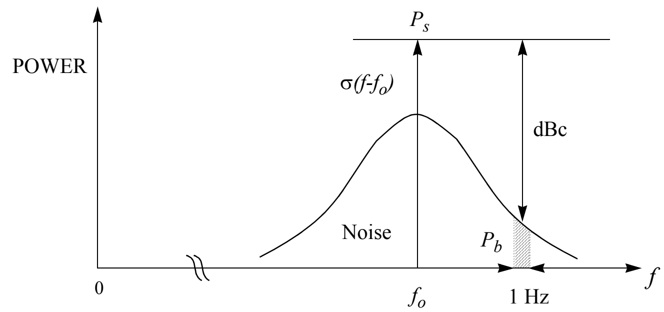

Figure 3

The relative level of the noise sidebands defines the spectral purity, usually expressed in dBc (dB from the carrier) $\mathrm{d} = 10\log\left(\frac{P_b(1\,\mathrm{Hz})}{P_s}\right)$

NOTE: The spectral purity takes into account all the disturbances: phase noise and amplitude noise. The usual measurement instrument is the spectrum Analyzer.

AMPLITUDE NOISE AND PHASE NOISE

A real signal perturbed by the above two types of noise can be represented by the expression

$$v(t) = A[1 + a(t)]\sin(2\pi f_o t + \phi(t))$$

where $\alpha(t)$ and $\phi(t)$ represent two random functions to which correspond spectral density $S_a(f)$ (units: Volt^2/Hz) and $S_\phi(f)$ (units: radians^2/Hz), respectively.

The function $\phi(t)$ represents the phase fluctuations around the theoretical phase of the signal $\phi_o = 2\pi f_o t$ as illustrated in Figure 4.

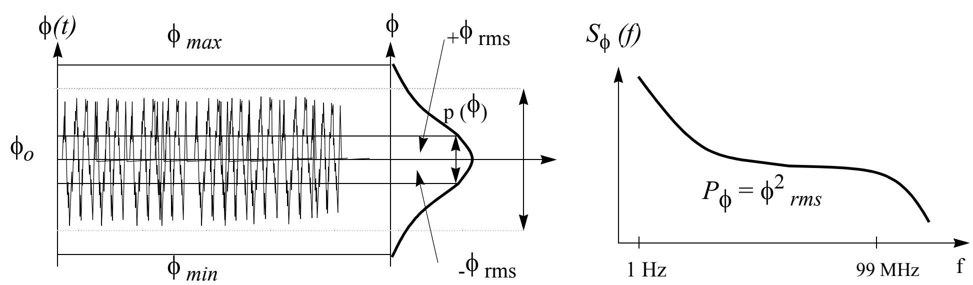

Figure 4

ϕ_{max} and ϕ_{min} represent the maximum deviation from the theoretical phase and enables the definition of a bit-error rate in digital phase modulation. For example, if the modulation has four phase states ($+\pi/4$, $+3\pi/4$, $-\pi/4$, $-3\pi/4$), the decision limit will be of $\pm\pi/4$ around each state. If the ϕ_{max} and ϕ_{min} values were lower than $\pm\pi/4$, then the phase noise would not induce transmission errors.

ϕ_{rms} represents the rms value of phase noise. It can also be called "mean phase fluctuation".

The frequency fluctuations of the carrier signal can be calculated as

$$f_o(t) = f_o + \frac{d\phi(t)}{2\pi dt} = f_o + \frac{\phi(t)}{2\pi} = f_o + \Delta f(t) \ \ Hz$$

where $\Delta f(t)$ represents the instantaneous fluctuations of the carrier frequency around its center value f_o. See Figure 5.

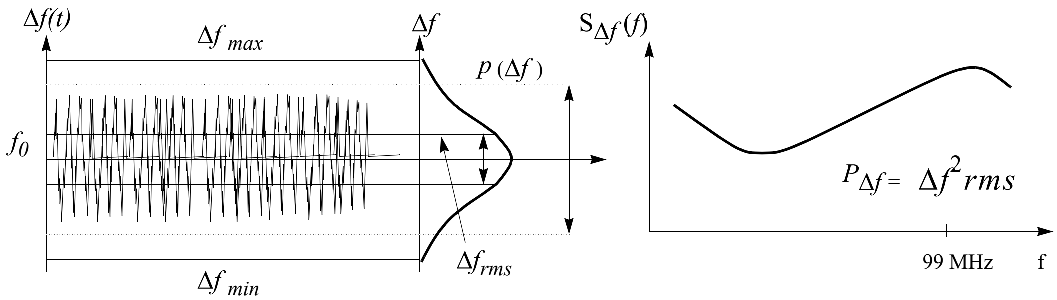

Figure 5

Δf_{rms} represents the rms value of the frequency noise (also referred to as mean frequency fluctuation).

The relationship between the spectral densities of phase and frequency fluctuations is

$$S_{\Delta f}(f) = f^2 S\phi(f)$$

Fractional frequency fluctuations can also be represented

$$y(t) = \frac{\Delta f(t)}{f_o}$$

With this representation, one is no longer dependent on the center frequency, fo,

The spectral density is

$$S_y(f) = f^2 / f_o^{\,2} S\phi(f)$$

The spectrums $S_\phi(f)$, $S_{\Delta f}(f)$, $S_y(f)$ represent the characterization of the frequency instability in the frequency domain. In industrial applications, the most commonly used spectrums are $S_a(f)$ and $S_\phi(f)$ because they provide an immediate estimation of spectral purity $S_v(f)$.

RELATIONSHIP BETWEEN SPECTRAL PURITY AND SPECTRAL DENSITY:

Assume the real signal $v(t) = A[1 + a(t)]\sin[2\pi f_o t + \phi(t)]$

If the amplitude *a(t)* and phase noise *φ(t)* are demodulated, the spectral analysis of these two random functions gives the following spectral densities $S_a(f)$ and $S_\phi(f)$. See Figure 6.

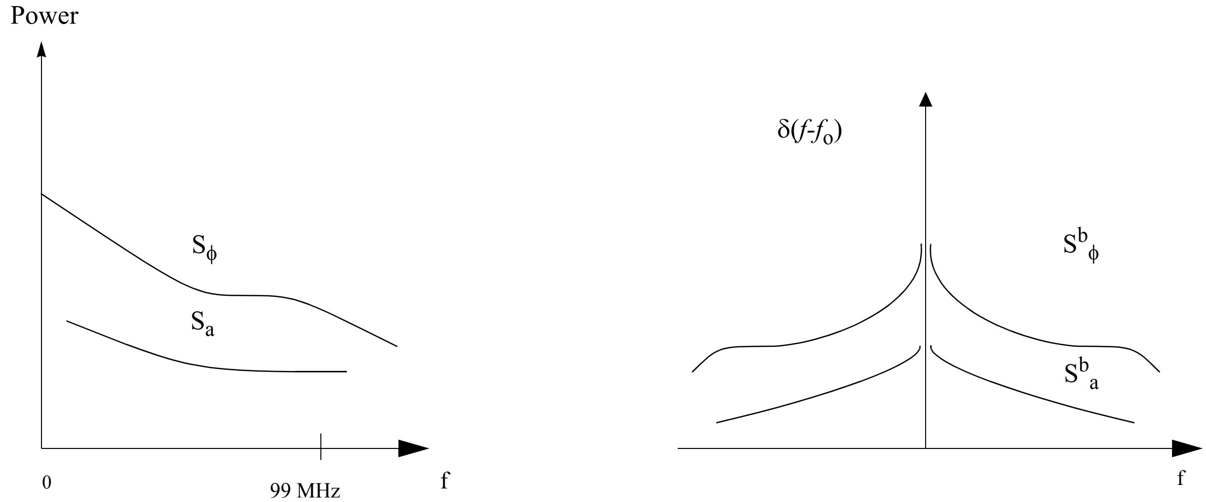

Figure 6

A useful approximation (phase noise power << 1 radian^2) is to assume that the sidebands that surround the carrier frequency f_o correspond to the double-band translation of the spectral densities $S_a(f)$ and

$S_\phi(f)$ --> $S_a^b(f)$ and $S_\phi^b(f) \quad \equiv S_\phi^b$.

If the approximation is not valid, the following exact relationship is obtained

$$Sv(f) = \frac{A^2}{2e^{-\overline{\phi^2}}}\left[\sigma(f-f_o) + S_a^b(f-f_o) + S_\phi^b(f-f_o) + \sum_n \frac{1}{n!}\, S_\phi^b(f) \otimes S_\phi^b(f)\right]$$

where $\otimes$ = convolution and $\overline{\phi^2}$ = total phase noise power

In many cases, particularly with microwave signals, one should take into account the first phase noise convolution $S_\phi^b(f) \otimes S_\phi^b(f)$ and the parameter $e^{-\overline{\phi^2}}$. The amplitude noise is usually low enough to only take into account $S_a(f)$.

TIME DOMAIN CHARACTERIZATION OF PHASE NOISE AND FREQUENCY NOISE

The frequency of the signal can be measured with a frequency counter. Frequency instability results in a fluctuation of the measurement results

$$\Delta f = f_o + \Delta f(t)$$

where t is the measuring time of the frequency counter (e.g. t = 1ms, 10ms, 100ms, etc.)

If the same measurement is made several times, the result will be a series of random data $\Delta f_n(T)$. The longer the measuring time t, the lower the $\Delta f_n(T)$ values.

If one does not take into account the drift of the carrier frequency (because it results from a different noise process) the average value of the $\Delta f_n(T)$ is zero.

The statistical analysis of these $\Delta f_n(T)$ deviations represent the characterization of frequency instability in the time domain, the fundamental parameter of which is the standard deviation or effective variance

$$\sigma[\Delta f(T)] = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\left[\Delta f_i(T) - \frac{1}{N}\left(\sum_{j=1}^{N}\Delta f_j(T)\right)\right]^2}\ Hz$$

Note that N must be > 100 Hz

The results are often normalized to the nominal frequency f_o

$$\sigma\frac{[\Delta f(1ms)]}{f_o} = 10^{-x}$$

If $\sigma\frac{[\Delta f(1ms)]}{f_o} = 10^{-7}$, and $f_o = 100$ MHz, then $\sigma(1\text{ ms}) = 10$ Hz, based on the peak value criterion $\Delta f_{max} = \pm 3 \times 10\text{ Hz} = \pm 30\text{ Hz}$.

RELATIONSHIP WITH THE INSTANTANEOUS FREQUENCY FLUCTUATIONS

At first glance, the two parameters seem similar. $\Delta f(t)$ represents the instantaneous frequency fluctuations, while $\Delta f(T)$ represents the frequency deviations over a time T, measured with a frequency counter.

The two parameters are related by the frequency noise power

$$\sigma[\Delta f(T)] = \left(\frac{1}{f_o}\right)\sqrt{P_{\Delta f}}$$

where $P_{\Delta f}$ = noise power in the bandwidth $\frac{\sin(\pi T f)}{(\pi T f)}$. This is equivalent to a low pass filter with a cutoff frequency of 0.25/T

$$P_{\Delta f} = \int_0^{\infty} S_{\Delta f}(f)\left[\frac{\sin(\pi T f)}{(\pi T f)}\right]^2 df$$

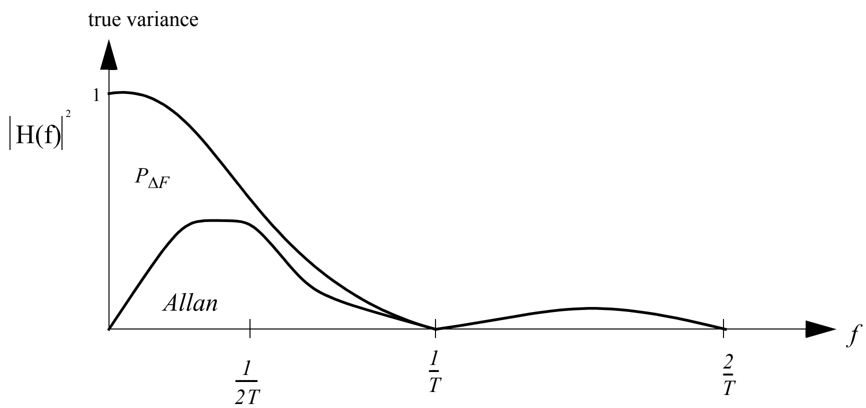

Figure 7

The frequency counter acts like a frequency noise integrator.

NOTE: A frequency spur at 1/T is not taken into consideration with this measurement, since it is suppressed by the zero value of the transfer function. For example, T = 1 ms, F = 1 kHz, T = 100 ms, F = 100 Hz.

Other variances have been introduced to simplify the statistical calculations. The Allan variance, for example, is based on only two successive measurements. The time variance, however, requires storage of all N measurements before calculation.

$$\sigma_{Allan}[\Delta f(T)] = \left(\frac{1}{N}\right)\sqrt{\sum_{i=1}^{N}\left(\frac{1}{2}\right)[\Delta f_i(T) - \Delta f_{i+1}(T)]^2}$$

Its transfer function $sin^2 (\pi T f)/(\pi T f)$ produces a greater selectivity (f_{max} = 0.37/T) and suppresses the contribution of the VLF (very low frequency) disturbances, and leads to results which will be different than those obtained with time variance calculations.

NOTE: The measurement results depend on the type of variance and the measurement time of the frequency counter

NOTE: These measurement processes do not characterize amplitude noise.

1.5 TIME NOISE OR JITTER

For digital transmission systems, it is productive to characterize the jitter of a signal . The phase fluctuations give rise to fluctuations of the zero crossing of the carrier signal

$$v(t) = A\sin[2\pi f_o t + \phi(t)] = A\sin\left[2\pi f_o\left(\frac{t+\phi(t)}{2\pi f_o}\right)\right]$$

$$v(t) = A\sin[2\pi f_o(t + \Delta T(t))]$$

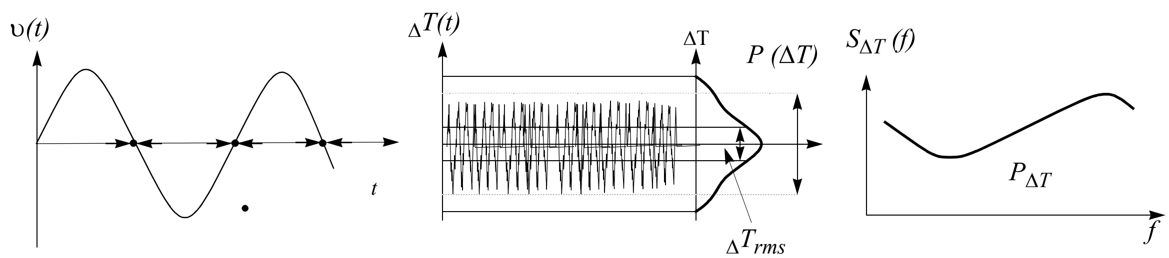

Figure 8

These time fluctuations are related to the phase fluctuations as follows

$$\Delta T(t) = \frac{\phi(t)}{2\pi f_o}$$

ΔT(t) is a random function, where the following parameters are defined:

- Distribution

  $p(\Delta T)$

- Mean Fluctuation

  ΔT_{rms} (standard deviation)

- Peak-to-peak Fluctuation

  $\pm 3 \Delta T_{rms}$

- Spectral Density

$$S_{\Delta T}(f) = \left(\frac{1}{2\pi f_o}\right) S_{\phi}(f) \qquad \text{seconds}^2/\text{Hz}$$

and where jitter power is $P_{\Delta T} = \int S_T(f)df$ seconds^2 and the Mean Fluctuation is $\Delta T_{rms} = \sqrt{P_{\Delta T}}$ seconds$_{rms}$

Jitter can be normalized to the period of the signal. The unit interval (UI) is then defined

$$UI(t) = \left[\frac{\Delta T(t)}{T_o}\right] = \left[\frac{\phi(t)}{2\pi}\right] \qquad \text{UI(t) is a unitless quantity}$$

One often considers that the peak-to-peak value of this parameter is the most interesting, since with this value a threshold of disturbance UI_{pp} can be defined.

2 MEASURING METHOD

The parameters to be measured are the following:

- Spectral Purity
- Amplitude Noise
- Phase Noise
- Frequency Noise
- Variances
- Jitter

2.1 SPECTRUM ANALYZER

The parameter measured by a spectrum analyzer is spectral purity. See Figure 9.

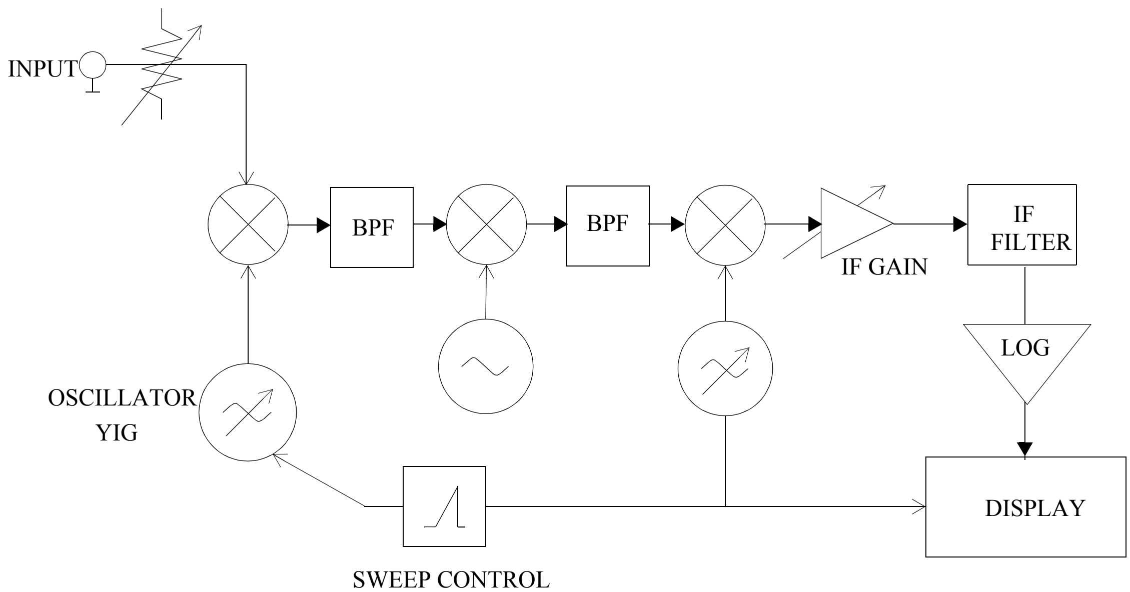

Figure 9

The principle is based on shifting the signal to be measured to an IF (intermediate frequency). The critical points are:

- Local oscillators
- IF filtering
- Dynamic range of the logarithmic detection

Advantages:

- Direct measurement at the signal frequency
- All the functions that are necessary for measurements are incorporated in the instrument (demodulation, references, etc.)
- Easy to use.

Disadvantages:

In spite of some recent improvements, such as synthesized oscillators and an increase of the "log-amplifier" dynamic range (80 --> 100 dB), when used specifically for noise measurement, this instrument has many disadvantages:

- The phase noise of the local oscillators is too high for most of the sources to be tested (often, the local oscillators are synchronized oscillators, not synthesized oscillators, so the mean value of the center frequency is stable but the noise remains very high).
- The instability of the center frequency makes it difficult to make measurements close to the carrier.
- It is not possible to differentiate spurious and noise since the spectrum shows only dBm values.

- The analysis is linear and related to the distance from the carrier, which implies a small analysis span (max = deviation, min = deviation / 50).
- The spectrum is not normalized as dB/carrier.
- There are many measurement inaccuracies (attenuator, gain, IF filter, bandwidth, amplitude-log linearity, etc.).
- Detection is linear and not quadratic.
- The fact that the video smoothing is operated after the log-amplification induces a 2.5 dB error on a Gaussian noise.
- The carrier signal limits the dynamic range.
- The input noise factor is considerable (20 dB).
- The results are not processed.

2.2 DEMODULATION OF AMPLITUDE AND PHASE NOISE

Amplitude Noise:

This function is easily carried out by a wide-band detection diode. An adaptation network is necessary to define the detection impedance and suppress the direct voltage which could overload the low-noise amplifier. Calibration is performed with a variable amplitude source.

Phase Noise:

By maintaining a reference source in quadrature, phase noise can be demodulated using a double balanced mixer. To maintain quadrature, a phase lock loop is usually used, in which a balanced mixer acts as a phase-sensitive detector (other types of detectors are used to perform phase control instead of quadrature control). See Figure 10.

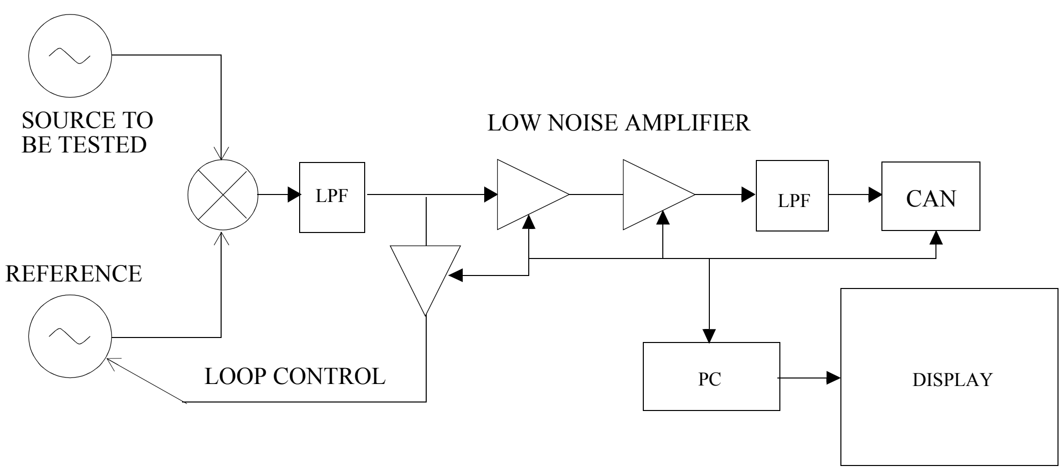

Figure 10

Disadvantages:

- The reference source must have lower noise than the source to be measured.
- The phase lock loop must be very precisely controlled, since it effects measurement results, its effect must be compensated for.

Advantages:

- Easy to set up and use, quick and precise calibration.
- Suppression of the carrier signal, which enables a wide measurement dynamic range.
- Log-log analysis, enabling a wide dynamic range in level (0-170 dB) and frequency deviation (1 Hz - 10 MHz)
- Substantial sensitivity (170 dB, residual only limited by the reference source)
- Measurements can be performed very close to the carrier
- Differentiation of spurious and noise. Noise in dB/Hz and spurious in dB
- Separate measurements of phase noise and amplitude noise
- Low sensitivity of measurements to the drift of the source to be tested (phase lock loop)
- Complete data processing (power of the computer associated with the Set)
- Automated measurements

2.3 DEMODULATION OF FREQUENCY NOISE

The lowest noise frequency demodulation can be obtained through a delay line (the other types of demodulation, e.g. diode demodulation, are not sensitive or stable enough for phase noise measurements). Figure 11 shows the measurement method.

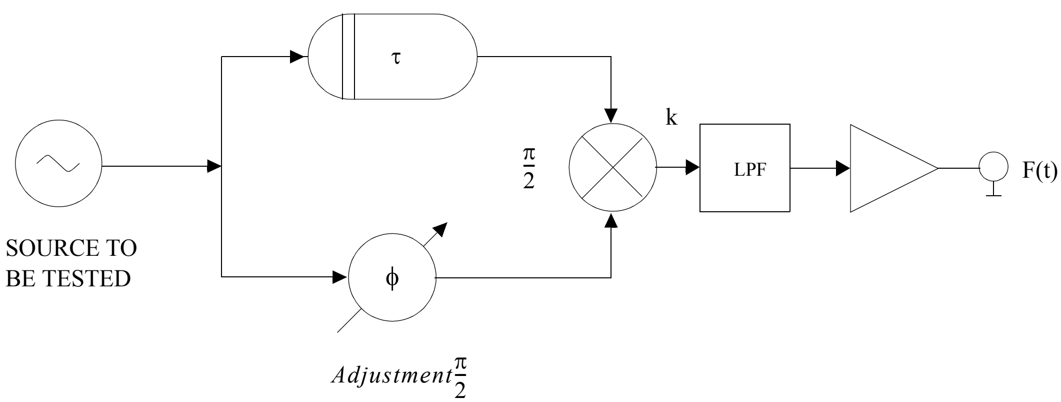

Figure 11

The delay line (τ), introduced into a channel of the balanced mixer, acts as a frequency discriminator, for frequencies << 1/τ.

The output of the mixer is a signal F(t), which is proportional to the frequency fluctuations:

$$F(t) = (KG2\pi T)\Delta f(t)\left[\frac{\sin(\pi f T)}{(\pi f T)}\right]$$

$$F(t) = (KG2\pi T)\Delta f(t) \text{for } f < \frac{1}{10}T$$

The demodulation factor depends on the value of the delay T.

Advantages:

- No reference source. It is difficult, however, to have a wide-band delay line (e.g. 1 GHz - 18 GHz). Therefore, frequency conversion is necessary to use the delay line at a fixed frequency, and this brings us back to the problem of the reference source.
- No phase control
- The data processing capability can be as high as it is for phase demodulation.

Disadvantages:

- Requires a reference source for wide-band operation.
- Complicated calibration process. The frequency has to be shifted to allow for calibration of the demodulation mixer.
- Difficult choice between a long delay to obtain increased demodulation sensitivity $K2\pi T$ and a short delay to get the large span spectral analysis (max. deviation << 1/T).

2.4 FREQUENCY COUNTING

The basic instrument is a frequency counter, which is used in its reciprocal mode for increased resolution. The parameters measured are the variances (true, Allan...). A reference source is necessary to obtain a residual noise compatible with the existing sources. Figure 12 shows the measurement method.

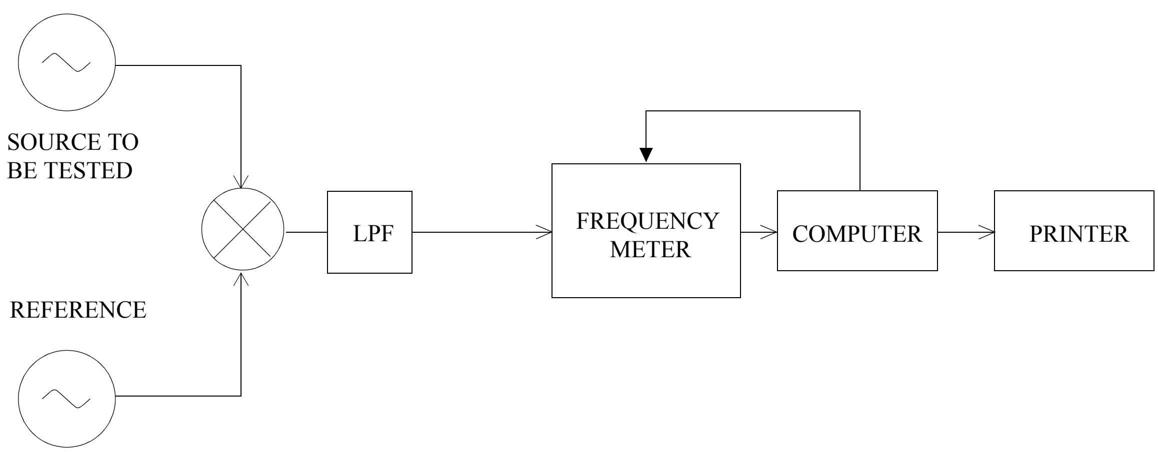

Figure 12

The measuring process is simple. Perform a statistical computation of the frequency measurement.

Advantages:

- The test set is easy to set up and use
- No calibration
- No phase lock loop required

Disadvantages:

- ‘The measurement time is long and measurements are taken serially
- The variances represent an integrated noise and the global values that result are therefore difficult to translate in terms of spectral purity

- The measurements are sensitive to spurious (because there are many zeros in the transfer function. Although this is not very important with regards to noise which shows a continuous spectrum, it is critical where spurious are concerned, since they can be overlooked depending on their frequencies)
- Needs a reference source

2.5 JITTER MEASUREMENT

The "eye pattern" method allows visualization of the phase margin between the modulation states. Figure 13 shows the measurement method.

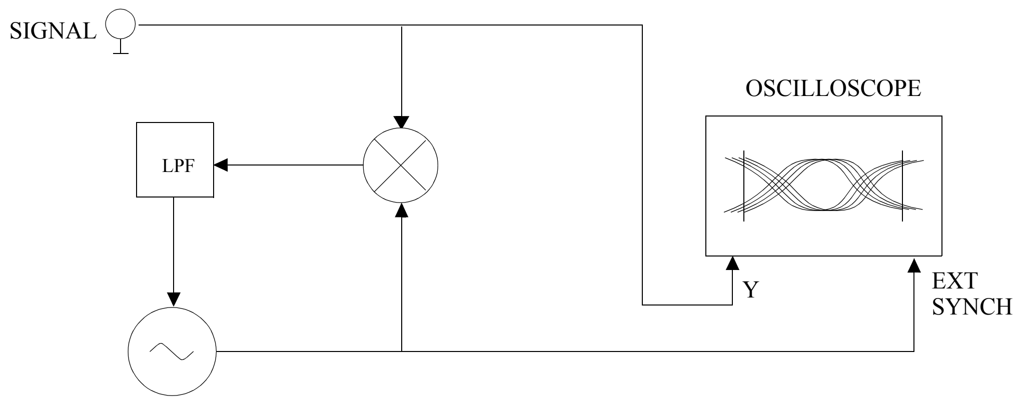

Figure 13

The demodulation method allows measurement of the peak-to-peak amplitude and the jitter spectral density, as shown in Figure 14.

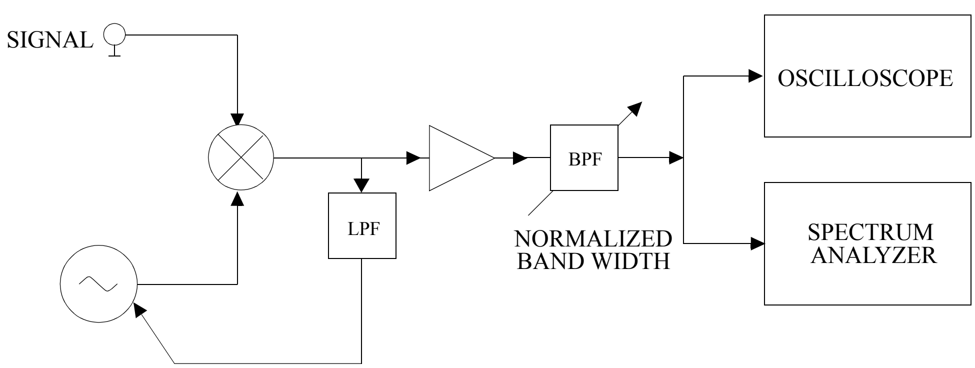

Figure 14

The resolution of this technique is low in comparison to the preceding methods, but sufficient to test contemporary digital links.

3 NOISE IN FREQUENCY GENERATORS

3.1 NOISE IN THE CIRCUITS

Noise sources can be separated into two categories, ultimate and excess, according to their origin:

Ultimate noise

Thermal Noise and Shot Noise are referred to as ultimate because they derive from the physics of materials, and do not depend on the quality of the components. They can never be suppressed; one can only optimize their action. These types of noise can also be expressed mathematically.

Excess noise

Flicker Noise and Popcorn Noise are referred to as excess because they depend on the quality of the components, often on the cleanness of their surfaces. The same manufacturing process can produce components with very different noise levels. There is no mathematical expression that describes these types of noise.

THERMAL NOISE

Thermal noise is a power source *e(t)*. Its spectral density is white noise, which is constant in relation to the frequency

$$S_b(f) = 4KTR \qquad \frac{Volts^2}{Hz}$$

where,

K = Boltzmann factor = 1.38 X 10^{-23} joule/degrees Kelvin
T = Absolute temperature (in degrees Kelvin)
R = Resistance value in ohms

Noise power

$$P_b = \int_o^B S_b(f)df = 4KTRB \qquad Volts^2$$

Noise level per bandwidth

$$\sqrt{\overline{e^2}} = \sqrt{4KTR} \qquad \frac{Volts_{rms}}{\sqrt{Hz}}$$

Noise level in bandwidth B

$$\sqrt{\overline{e^2}} = \sqrt{4KTRB} \qquad Volts_{rms}$$

For example, if R = 1KΩ, then

$$\sqrt{\overline{e^2}} = \frac{4nV}{\sqrt{Hz}}$$

and in a 1MHz bandwidth

$$\sqrt{\overline{e^2}} = 4\mu V$$

Noise power transmitted to a load having the same impedance

$$P_a = \frac{P_b}{4R} = KTB \text{ watts}$$

P_a expressed in dBm, P_a = -174 dBm for a bandwidth of 1 Hz.

RELATIONSHIP BETWEEN SIGNAL AND NOISE

The following is for a 1Hz BW only:

- For a signal of 0 dBm, S/N =174 dB
- For a signal of +10 dBm, S/N=184 dB

SHOT NOISE

Shot Noise is modeled as a current source *i(t)*. Its spectral density is a white noise, $S_i(f) = 2\,q\,I\ Amps^2/Hz$

q is the charge on the electron = 1.602×10^{-19} Coulombs

- I is the direct current DC in Amperes

Noise Power $P_b = 2\,q\,I\,B$

Noise Current per unit bandwidth

$$\sqrt{\overline{i^2}} = \sqrt{2qI}\ \frac{Amps_{rms}}{\sqrt{Hz}}$$

For example, I = 1 mA

$$\sqrt{\overline{i^2}} = 18 \times 10^{-12} \frac{Amps_{rms}}{\sqrt{Hz}}$$

With an impedance of 1KΩ $\sqrt{\overline{v^2}} = R\sqrt{\overline{i^2}} = 18 \times 10^{-9} \frac{V_{rms}}{\sqrt{Hz}}$ compared to thermal noise $\sqrt{\overline{e^2}} = 4 \times 10^{-9} \frac{V_{rms}}{\sqrt{Hz}} = 1k\Omega$

FLICKER NOISE

This noise can be represented as either a voltage or a current source. It is found in all components, and is characterized by its spectral density variation slope f^{-1}

The slope of this spectrum is often expressed in dB per decade: 10 db/decade, or in dB per octave: 3 dB/Octave. With a Log-Log representation, it is easy to recognize this type of noise.

There is at present no physical theory to explain the mechanism of flicker noise.

POPCORN NOISE

This type of noise varies in time through random quantified jumps, which generate a VLF (very low frequency) spectral density. When this type of noise is present in a component, it can be concluded that this component has a major defect, and it must be eliminated or reduced by testing and selection.

SPECTRAL DENSITY OF THE NOISE OF ELECTRONIC CIRCUITS

To estimate the total noise of a circuit, one must add all the noise sources that have been described above (in power or spectral density). For example,

$$S_{total}(f) = 4KT(R_1 + R_2 + ...) + 2q(I_1R_1^2 + I_2R_2^2 + \quad ...) + \frac{K_1}{f} + \frac{K_2}{f} + ...$$

If the noise level has not been filtered, the noise spectral density will have the general outlook of white noise or flicker noise

$$S(f) = K_1 + \frac{K_2}{f}$$

Refer to figure 15:

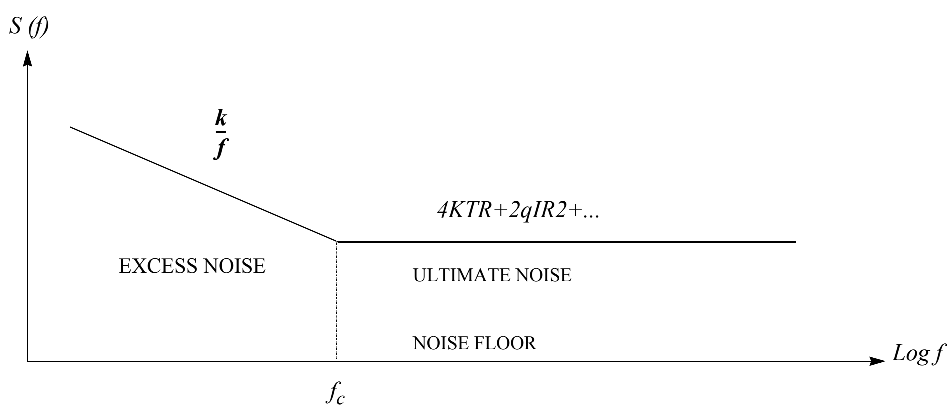

Figure 15

A cut-off frequency f_c called “flicker” cut-off frequency can be defined as,

$$S(f) = K\left[1 + \left(\frac{f_c}{f}\right)\right]$$

In silicon technology: $1KHz \le f_c \le 10kHz$

In GaAs technology: $f_c \approx 100MHz$

3.2 NOISE IN OSCILLATORS

An oscillator can be modeled as an amplifier with a band-pass filter, as shown in Figure 16

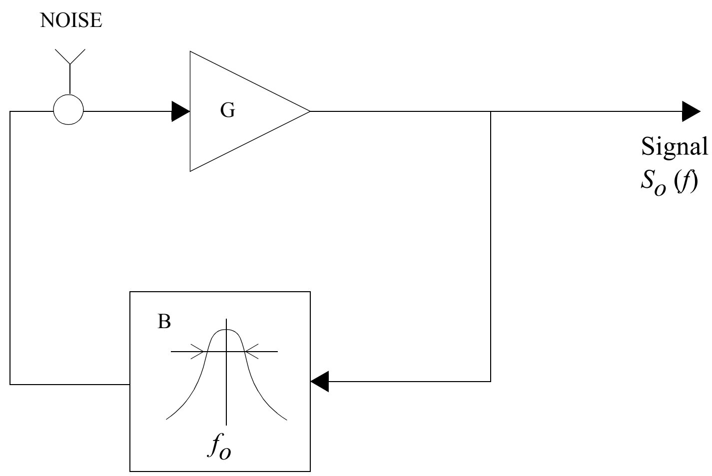

Figure 16

The filter is defined by its center frequency f_o and its band-pass B, as a function of its quality coefficient Q

$$B = \frac{f_o}{Q}$$

Three types of noise will disturb the signal:

1. The amplifier's noise sources generate phase noise $\phi(t)$.
2. Since the center frequency f_o is defined by the resonator, the parametric noise (variation of a parameter which defines the f_o value) modulates the oscillator by generating a frequency noise

$$f_o(t) = f_o + \Delta f(t)$$

3. If the frequency of the oscillator can be controlled by an external input, a noise signal applied to this input modulates the oscillator in frequency, thus generating frequency noise.

NOISE GENERATED BY THE AMPLIFIER

The study of circuit noise has shown us that the spectral density of noise sources can be represented by

$$S_b(f) = K1\left[1 + \left(\frac{f_c}{f}\right)\right]$$

comprising white noise and flicker noise with units of Volt^2/Hz.

This noise modulates the signal's phase going through the amplifier and the phase noise which is generated depends on the level (A) of the signal

$$S_\phi(f) = \left(\frac{K_1}{2A^2}\right)\left[1 + \frac{f_c}{f}\right] \qquad \frac{\text{radian}^2}{\text{Hz}}$$

The resonator filters the noise with the following transfer function

$$[H(f)]^2 = \left[1 + \left(\frac{f_o}{2Qf}\right)^2\right]$$

resulting in the phase noise spectral density

$$S_o(f) = S_\phi(f) \times [H(f)]^2$$

and thus:

$$S_o(f) = \left(\frac{K1}{2A^2}\right)\left[1 + \frac{f_c}{f}\right] \times \left[1 + \left(\frac{f_o}{2Qf}\right)^2\right]$$

Two major cases are to be found in most oscillators, directly related to their quality coefficient:

1. High Q coefficient oscillators, such as $\left(\frac{f_o}{2Q}\right) \leq f_c$. For example, a crystal oscillator: f_o = 10 MHz, Q = 10,000, f_c = 1 kHz

$$\frac{f_o}{2Q} = 5Hz \ll f_c = 1KHz$$

2. Low quality coefficient or high center frequency f_o, such as $\left(\frac{f_o}{2Q}\right) > f_c$ for example, a DRO: f_o = 3 GHz, Q = 1000, and f_c = 10 kHz

$$\frac{f_o}{2Q} = 1.5MHz \gg f_c = 10KHz$$

Figure 17 shows the spectral densities of the phase fluctuations obtained in both of the above cases.

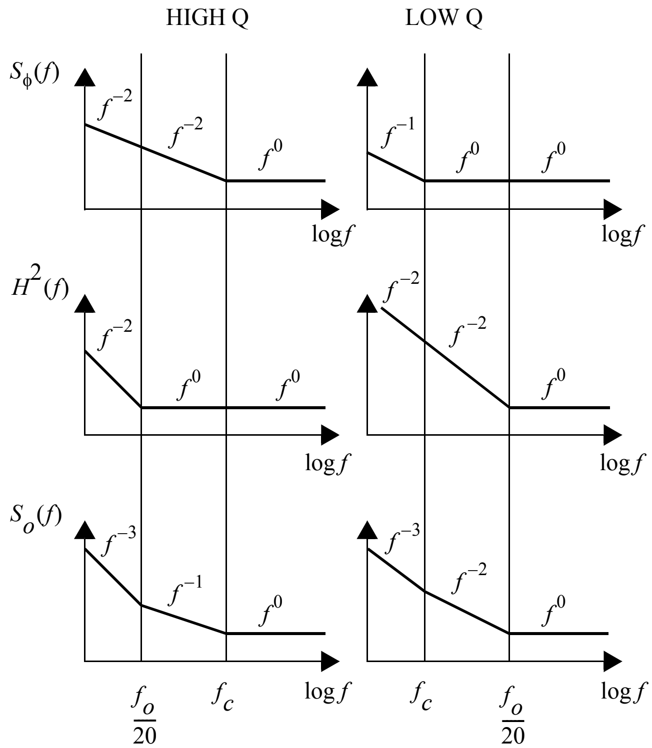

Figure 17

1st case $\left(\frac{f_o}{2Q}\right) < f_c$ The following rules are determined:

- *1/f* ^{3}- flicker frequency noise, 30 dB / decade or 9 dB / Octave
- *1/f* - flicker frequency noise, 10 dB / decade or 3 dB / Octave
- *f* - white phase noise, called the noise floor

2nd case: $\left(\frac{f_o}{2Q}\right) > f_c$ The following rules are obtained:

- *1/f* ^{3} - flicker frequency noise, 30 dB / decade or 9 dB / Octave
- *1/f* ^{2} - blank frequency noise, 20 dB / decade or 6 dB / Octave
- *f* - white phase noise, called the noise floor

A Log-Log representation is absolutely necessary to recognize these different slopes. The study of these slopes enables the analysis of the oscillator noise:

- loaded quality coefficient
- oscillation level
- amplifier noise

The main difficulty with this analysis lies in the fact that this type of noise can be mistaken for the parametric and frequency control noise sources.

PARAMETRIC NOISE

Some of the components of the band-pass filter such as the varicaps, crystals, trimming capacitors, etc. have fluctuations which directly affect the oscillating frequency. Assume a fluctuation of the capacitor given by

$$2\pi fo = \left(\frac{L}{C}\right)^{\frac{1}{2}}$$

$$\text{if } C = C_o + C(t) \rightarrow f_o(t) = f_o + \Delta f(t)$$

where $\Delta f(t)$ is the frequency noise introduced by $C(t)$.

This is a parametric effect: the level of the noise is not related to the oscillation level, filter Q, or the amplifier noise. Generally, this frequency noise has a flicker type spectral density

$$S_{\Delta f}(f) = \frac{K}{f}$$

i.e. a phase noise

$$S\phi(f) = \frac{1}{f^2}$$

$$S_{\Delta f}(f) = \frac{K}{f^3}$$

This noise merges with the "flicker" frequency noise given by the amplifier, which has the same slope. This type of noise is also characterized by a substantial variation from one component to another. For example, a 10 MHz crystal oscillator has a noise level between -115 dBc and -135 dBc at 10 Hz offset from the carrier, depending on its manufacturing process. However, it can vary by as much as 10 dB between the crystals that are produced by the same manufacturing process.

OSCILLATOR CONTROL NOISE

If noise is added to the control voltage, it will modulate the oscillator, generating a frequency noise.

If the slope of the oscillator control is K Hz/Volt, $b(t)$ is the voltage noise input with a spectral density $S_b(f)$. The following is obtained:

- the frequency fluctuations: $\Delta f(t) = K\, b\,(t)$

- The spectral density of these fluctuations: $S_{\Delta f}(f) = K^2 S_b(f)$
- The spectral density of phase noise $S_\phi(f) = \frac{K^2}{f^2} S_b(f)$

NOTE: $S\phi(f) = S_b(f)$ for $f = K$

For example, an oscillator with a 1 MHz/Volt slope and a control resistance of 1KΩ.

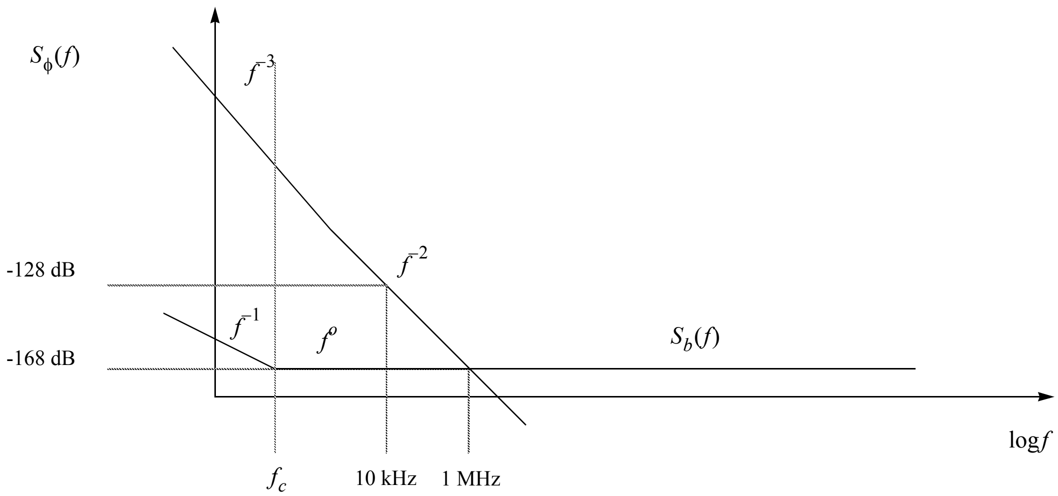

Figure 18

The noise level generated by the resistance is $\sqrt{\overline{b^2}} = 4x10^{-9} V_{rms}/\sqrt{Hz}$

The noise spectral density is $S_b(f) = 16x10^{-18}\frac{Volt^2}{Hz} = 168dB_{volt}$

The phase noise spectral density is $S_\phi(f) = \frac{K^2}{f^2} S_b(f)$

$$S\phi(f) = (1 \times 10^6)^2 \times \frac{16 \times 10^{-18}}{f^2}$$

$$S\phi(f) = \frac{1.6 \times 10^{-5}}{f^2}$$

At 10 KHz, S_ϕ (10 KHz) = 1.6 X 10^{-13}, corresponding to: S_ϕ (10 KHz) = -128 dB

Spectral purity S_v (10 KHz) = -128 dB - 3 dB = -131 dB

If the control noise is white ($f > f_C$, Figure 18), the phase noise obtained is white frequency noise with a slope of 20 dB per decade.

If the control noise is flicker ($f < f_C$, Figure 18), the phase noise obtained is a flicker frequency noise with a slope of 30 dB per decade.

3.3 NOISE IN THE PHASE LOCK LOOP

We shall investigate the phase noise spectral density of a Voltage Controlled Oscillator (VCO) locked to a high stability reference. Figure 19 shows a simplified model of a phase lock loop.

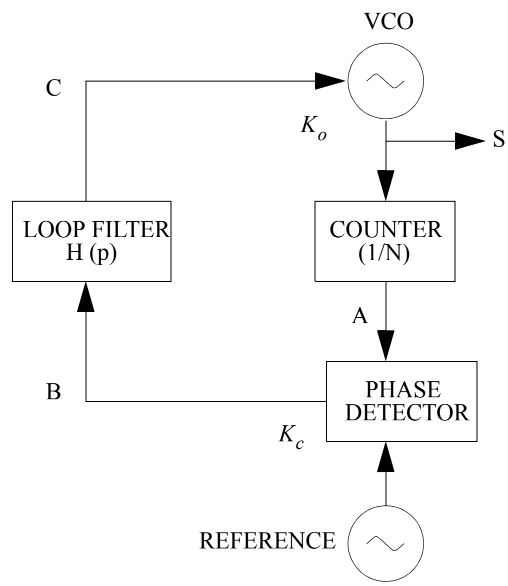

Figure 19

where (p) is the frequency domain, (Laplace notation) (= jw)

K_c is the gain of the phase comparator (Volt/Radian)

K_o is the slope of the VCO (Hz/Volt)

N is the dividing factor

$\phi_O(t)$ is the noise of the free running VCO

$\phi_S(t)$ is the noise of the controlled VCO

$\phi_R(t)$ is the noise of the reference source

The phase fluctuations of the VCO are divided by the counter N

$$\phi_a(p) = \frac{1}{N\phi_s(p)}$$

The output level of the phase comparator is proportional to the difference in phase of the two channels

$$V_b(p) = K_c\left[\phi_R(p) - \frac{\phi_s(p)}{N}\right]$$

The output level is filtered and applied to the varicap of the VCO's frequency control:

$$V_c(p) = H(p)K_c\left[\phi_R(p) - \frac{\phi_s(p)}{N}\right]$$

$$\phi_{vco}(p) = K_o V_c(p) = \left(K_o K_c \frac{H(p)}{p}\right)\left[\phi_R(p) - \frac{\phi_s(p)}{N}\right]$$

The VCO adds its internal noise:

$$\phi_{vco}(p) = \left(K_o K_c \frac{H(p)}{p}\right)\left[\phi_R(p) - \frac{\phi_s(p)}{N}\right] + \phi_o(p)$$

Because the system is locked, the fluctuations of the controlled VCO ($\phi_S\,(p)$) are also present

$$\phi_s(p) = K_o K_c \frac{H(p)}{p}\left[\phi_R(p) - \frac{\phi_s(p)}{N}\right] + \phi_o(p)$$

Assume that $K = \dfrac{K_o K_c}{N}$

$$\phi_s(p) = \left[\frac{p}{(p + KH(p))}\right]\phi_o(p) + \left[\frac{NKH(p)}{(p + KH(p))}\right]\phi_R(p)$$

$$\phi_s(p) = H_o(p)\phi_o(p) + NH_R(p)\phi_R(p)$$

Or, expressed in terms of spectral density

$$S_{\phi_s}(f) = |H_o(f)|^2 S_{\phi_o}(f) + N^2|H_R(f)|^2 S_{\phi_R}(f)$$

To obtain a second-order loop (proportional and integral gain), the standard transfer functions are

$$|H_o(f)|^2 = \left[\frac{(4f_n^2\xi^2 f^2 + f_n^4)}{f^4 + (4f_n^2\xi^2 - 2f_n^2)f^2 + f_n^4)}\right]$$

$$|H_R(f)|^2 = \left[\frac{f^4}{(f^4 + (4f_n^2\xi^2 - 2f_n^2)f^2 + f_n^4)}\right]$$

where :

f_n is the “natural” cut-off frequency of the loop $\sqrt{\dfrac{G}{\tau_{RC}}}$

G is the DC loop gain $\dfrac{K_c K_o H(O)}{N}$

τ_{RC} is the integrator time constant value $\xi_x = 2\pi RC$

Where:

ξ is the loop damping factor $\left(\dfrac{1}{2}\right)\sqrt{\dfrac{f_b}{f_{RC}}}$

f_b is the cut-off frequency of the continuous gain: G

f_{RC} is the cut-off frequency of the integral loop: $\dfrac{1}{\tau_{RC}}$

f_n is the geometric average of the two cut-off frequencies f_b and f_{RC}

$$f_n^2 = \frac{G}{\tau_{RC}} = G \times f_{RC} = f_b \times f_{RC}$$

The transfer function $H_R(f)$ corresponds to a low-pass filtering of the reference noise multiplied by N. See Figure 20.

The transfer function $H_o(f)$ corresponds to a high-pass filtering of the VCO noise. See Figure 20.

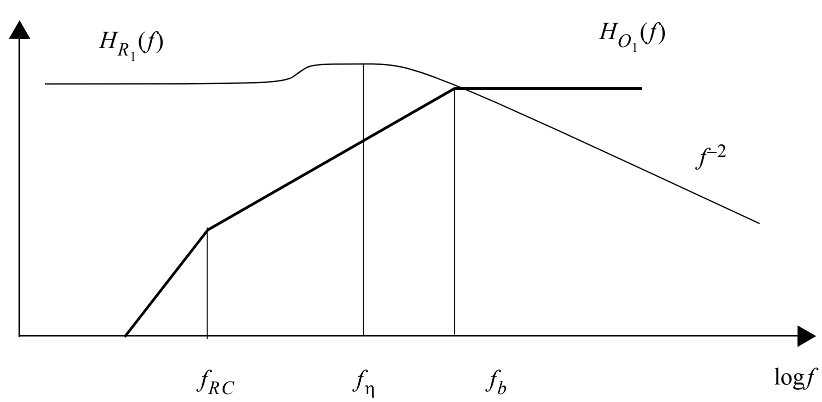

Figure 20

The spectral density of the output signal, Figure 21, copies the noise of the reference multiplied by N (or N^2 in terms of spectral density) close to the carrier, and copies the noise of the free-running VCO far from the carrier. Between these two influences a noise platform is often observed, which arises either from the noise of the VCO filtered by the loop (shown in Figure 21) or from the noise of the phase comparator, or the divider.

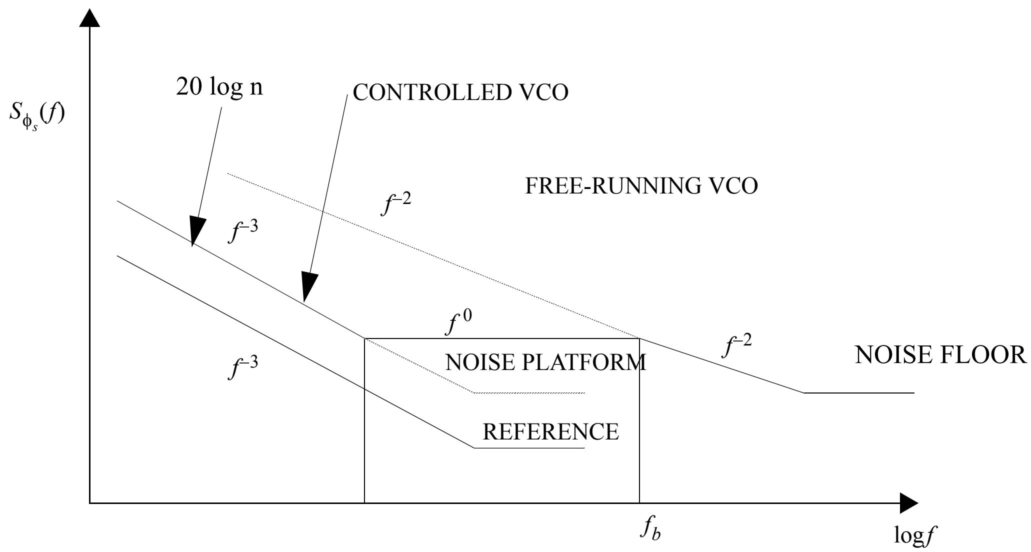

Figure 21

4 CHARACTERIZATION OF FREQUENCY GENERATORS

The noise of a frequency generator is characterized mainly by the spectral density of its phase noise $S_\phi(f)$ and amplitude noise $S_a(f)$.

This representation is useful because it enables calculation, on the basis of these spectrums, of all the other parameters which characterize the noise of a frequency generator.

Take the spectrums, shown in Figure 22, for a carrier frequency of 10 MHz. The amplitude noise and phase noise have the same noise floor: -140 dB.

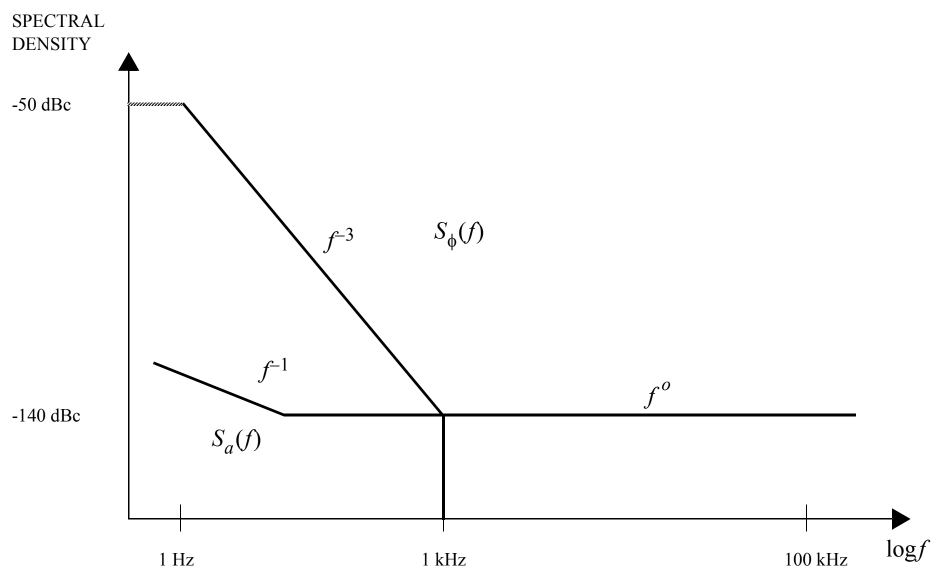

Figure 22

4.1 SPECTRAL PURITY

Spectral purity is expressed in dBc. It is the relation between the power of the signal P_s and the total noise power P_b, per unit bandwidth (1 Hz), and at a frequency f_m from the carrier

$$dBc = 10\log\left(\frac{P_b 1Hz}{P_s}\right)$$

The Phase Noise contribution is called $L(f)$. A first approximation gives

$$L(f) = \frac{1}{2}S_{\phi}(f) \text{which results in} L(f)dBc = S_{\phi}(f)dB - 3dB$$

example: Noise floor of *L(f)* = -140 dB -3 dB = -143 dBc

The Amplitude Noise contribution is called *M(f)*

$$M(f) = \frac{1}{2}S_{a}(f) \text{which results in} M(f)dBc = S_{a}(f)dB - 3dB$$

example: Noise floor of *M(f)* = -140 dB -3 dB = -143 dBc

The spectrum at the carrier frequency fo is $S_v(f) = L(f) + M(f)$

example: Noise floor of $S_v(f)$ = -143 dBc -143 dBc = -140 dBc

This spectrum is symmetrical around the carrier frequency (two-sided)

*Spectral Densities immediately become spectral purity, if one considers that: phase noise << 1 radian^2. (The complete expression is given in Section 1.)

4.2 PHASE NOISE POWER

By integrating the phase noise spectral density $S\phi(f)$ into a bandpass B, the noise power obtained can be represented as

$$P\phi = \int_{o}^{B} S\phi(f)df \ \ radians^2$$

For the example presented in Figure 22

1 Hz-100 kHz: $P\phi = 5.54\ \mu\,rad^2$
1 Hz-1 kHz: $P\phi = 5.54\ \mu\,rad^2$
1 kHz-100 kHz: $P\phi = 0.001\ \mu\,rad^2$

The square root of this power gives the rms power of phase noise, which corresponds to the mean fluctuation of the signal's phase due to the noise:

$$\phi_{rms} = \sqrt{P\phi} \ \ radian_{rms}$$

1 Hz-100 kHz: $\phi_{rms} = 2.35\ m\,rad_{rms}$
1 Hz-1 kHz: $\phi_{rms} = 2.35\ m\,rad_{rms}$
1 kHz-100 kHz: $\phi_{rms} = 0.032\ m\,rad_{rms}$

Multiplying this value by 6 gives us the peak-to-peak (pp) fluctuation of this phase:

$$\phi_{pp} = 6\sqrt{P\phi} \;\; radian_{pp}$$

1 Hz-100 kHz: ϕ_{pp} = 14 m rad_{pp}

By calculating 10 log (Pϕ / 1 $radian^2$) the noise power obtained is in dB:

example Figure 22: 1 Hz - 100 kHz: Pϕ = -52.6 dB
1 Hz - 1 kHz: Pϕ = -52.6 dB
1 kHz - 100 kHz: Pϕ = -90 dB

NOTE: The phase noise power and mean phase fluctuation depend (in this example) on the lower part of the spectrum (f < 1 kHz), i.e. closer to the carrier.

4.3 FREQUENCY NOISE POWER

By multiplying the phase noise spectral density by the frequency square, one gets the frequency noise spectral density

$$S_{\Delta f}(f) = f^2 S\phi(f) \quad \frac{Hertz^2}{Hertz} = Hertz$$

Thus, the frequency noise power can be calculated

$$P_{\Delta f} = \int S_{\Delta f}(f)df = \int f^2 S\phi(f)df \quad Hertz^2$$

1 Hz-100 kHz: $P_{\Delta f}$ = 3.35 Hz^2

1 Hz-1 kHz: $P_{\Delta f}$ = 80 μHz^2

1 kHz-100 kHz: $P_{\Delta f}$ = 3.35 Hz^2

The square root of this power gives the mean fluctuation of the frequency of the signal, around its center value f_o

$$\Delta f_{rms} = \sqrt{P_{\Delta f}} \;\; Hertz_{rms}$$

1 Hz-100 kHz: Δf_{rms} = 1.83 Hz_{rms}

1 Hz-100 kHz: Δf_{rms} =8.95 m Hz_{rms}

1 kHz-100 kHz: Δf_{rms} = 1.83 Hz_{rms}

Multiplying this value by 6 gives us the peak-to-peak (pp) frequency fluctuation

$$\Delta f_{pp} = 6\sqrt{P_{\Delta f}} \;\; Hertz_{pp}$$

1 Hz-100 kHz: $\Delta f_{pp} = 11\ Hz_{pp}$

This value represents the maximum fluctuation of the center frequency, due to the noise.

NOTE: The frequency noise power and mean frequency fluctuation depend (in this example) on the higher part of the spectrum (f > 1 kHz), i.e. far from the carrier.

4.4 FREQUENCY STABILITY

The stability of the center frequency of the signal can be characterized by the “variance” of the measurement results obtained with a frequency counter. This measurement method is one of the oldest used to characterize frequency stability. The main interest of this method was that it was easy to set up. All the different types of “variance” can be calculated on the basis of the phase noise spectral density

“True variance”

$$\frac{\sigma[\Delta f(t)]}{f_o} = \left(\frac{1}{\pi f_o T}\right)\sqrt{\int_0^\infty S\phi(f)\sin^2(\pi T f)df}$$

“Allan variance”

$$\frac{\sigma[\Delta f(t)]}{f_o} = \left(\frac{2}{\pi f_o T}\right)\sqrt{\int_0^\infty S\phi(f)\sin^4(\pi T f)df}$$

The above expressions correspond to a noise power calculation in the bandwidths:

$\sin^2(\pi T f)$ and $\sin^4(\pi T f)$ (T = Time of measure)

The values are expressed as a function of time of measurement T. See Figure 23.

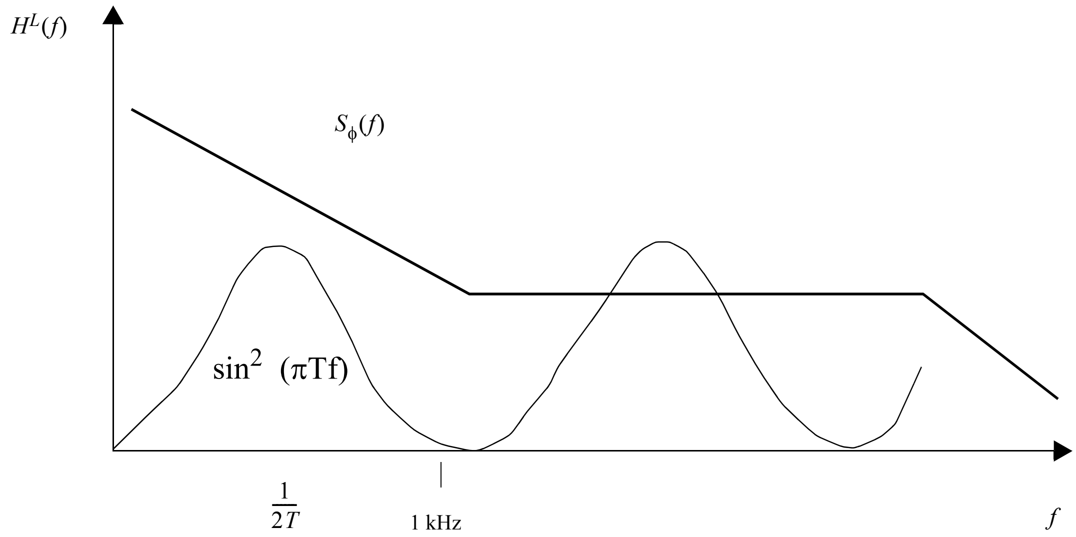

Figure 23

Example figure 22:

True Variance: (T=1ms) = 0.77 x 10^{-9}

Allan variance: (T=1ms) = 0.95 x 10^{-9}

The true variance corresponds to low-pass filtering of the frequency fluctuations, with a cut-off frequency f_c = 0.25/T. If this noise power is calculated with an ideal low-pass filter with f_c = 0.25/1 ms = 250 Hz, the following result is obtained

$$\Delta f_{rms} = 7.9 mHz_{rms}\text{, i.e.}\frac{\Delta f_{rms}}{f_o} = 0.79X10^{-9}$$

This result is very close to the calculated true variance: 0.77 X 10^{-9}

4.5 JITTER

Jitter, or "mean fluctuation" of the edge of the signal due to phase noise, is given by the relation:

$$\Delta T_{rms} = \left(\frac{1}{2\pi f_o}\right)\sqrt{\int_0^\infty S\phi(f)df} \quad \text{seconds}_{rms}$$

example Figure 22:

1 Hz-100 kHz: fo = 10 MHz : ΔT_{rms} = 37.5 picoseconds_{rms}

1 Hz-1 kHz: ΔT_{rms} = 37.5 picoseconds_{rms}

1 kHz-100 kHz: ΔT_{rms} = 0.5 picoseconds_{rms}

Normalized Jitter: UI_{pp} (unit interval peak-to-peak)

$$UI_{pp} = 6 \times \left[\frac{\Delta T_{rms}}{T_o}\right] = 6 \times \left[\frac{1}{2\pi}\sqrt{f_{\phi}(f) df}\right]$$

example Figure 22:

1 Hz - 100 KHz: $UI_{pp} = 2.25\ 10^{-3}$

The jitter depends (in this example) on the low-frequency fluctuations (f < 1 kHz), of the phase noise, i.e. close to the carrier

5 CONCLUSION

- Today's noise measurement instruments, which measure the noise on a carrier signal, are based on phase and amplitude demodulation.
- ‘This method is quick and easy to set up and allows the user to make precise and high resolution measurements.
- Processing the signal on phase noise and amplitude gives access to all the parameters which characterize a signal (spectral purity, noise power, frequency instability, jitter.)
- This method requires a reference source, as in fact, all noise measurement methods do.

CHINA
Tel: [+86] (10) 6467 2823
Fax: [+86] (10) 6467 2821

EUROPE
Tel: [+44] (0) 1438 742200
Fax: [+44] (0) 1438 727601

FRANCE
Tel: [+33] 1 60 79 96 00
Fax: [+33] 1 60 77 69 22

HONG KONG
Tel: [+852] 2832 7988
Fax: [+852] 2834 5364

SCANDINAVIA
Tel: [+45] 9614 0045
Fax: [+45] 9614 0047

SPAIN
Tel: [+44] (91) 640 11 34
Fax: [+44] (91) 640 06 40

UNITED KINGDOM
Tel: [+44] (0) 1438 742200
Toll Free: [+44] (0) 1438 742200 (UK only)
Fax: [+44] (0) 1438 727601

USA
Tel: [+1] (0) (316) 522 4981
Toll Free: [+1] (800) 835 2352 (US only)
Fax: [+1] (316) 522 1360

AEROFLEX
A passion for performance.

Our passion for performance is defined by three attributes represented by these three icons: solution-minded, performance-driven and customer-focused

www.aeroflex.com
info-test@aeroflex.com